THE NATURE OF PHILOSOPHICAL PROBLEMS AND THEIR ROOTS IN SCIENCE *

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I

It was after some hesitation that I decided to take as my point of departure the present position of English philosophy. For I believe that the function of a scientist or of a philosopher is to solve scientific or philosophical problems, rather than to talk about what he or other philosophers are doing or might do. Even an unsuccessful attempt to solve a scientific or philosophical problem, if it is an honest and devoted attempt, appears to me more significant than any discussion of a question such as 'What is science?' or 'What is philosophy?'. And even if we put this latter question, as we should, in the somewhat improved form 'What is the character of philosophical problems?', I for one should not bother much about it; I should feel that it has little weight if compared with even such a minor problem of philosophy, as, say, the question whether every discussion must always proceed from 'assumptions' or 'suppositions' which themselves are beyond argument.[1]

When describing 'What is the character of philosophical problems?' as a somewhat improved form of 'What is philosophy?', I wished to hint at one of the reasons for the futility of the current controversy concerning the nature of philosophy—the naive belief that there is an entity such as 'philosophy', or perhaps 'philosophical activity', and that it has a certain character or 'nature'. The belief that there is such a thing as physics, or biology, or archaeology, and that these 'studies' or 'disciplines' are distinguishable by the subject matter which they investigate, appears to me to be a residue from the time when one believed that a theory had to proceed from a definition of its own subject matter.[2] But subject matter, or kinds of things, or classes

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[1] I call this a minor problem because I believe that it can easily be solved, by refuting the ('relativistic') doctrine indicated in the text.

[2] This view is part of what I have called 'essentialism'. Cf. for example my *Open Society*, ch. 11, or 'The Poverty of Historicism I' (*Economica* N.S., 1944, **11**, No. 42).

of things, do not, I hold, constitute a basis for the distinction of disciplines. Disciplines are distinguished partly for historical reasons and reasons of administrative convenience (such as the organisation of teaching and of appointments), partly because the theories which we construct to solve our problems have a tendency [1] to grow into unified systems. But all this classification and distinction is a comparatively unimportant and superficial affair. *We are not students of subject matter but students of problems.* And problems may cut right across the borders of any subject matter or discipline.

Obvious as this fact may appear to some people, it is so important for our present discussion that it is worth while to illustrate it by an example. It hardly needs mentioning that a problem posed to a geologist such as the assessment of the chances of finding deposits of oil or of uranium in a certain district needs for its solution the help of theories and techniques usually classified as mathematical, physical, and chemical. It is, however, less obvious that even a more 'basic' science such as atomic physics may have to make use of a geological survey, and of geological theories and techniques, if it wishes to solve a problem arising in one of its most abstract and fundamental theories; for example, the problem of testing predictions concerning the relative stability or instability of atoms of an even or odd atomic number.

I am quite ready to admit that many problems, even if their solution involves the most diverse disciplines, nevertheless 'belong', in some sense, to one or another of the traditional disciplines; for example, the two problems mentioned 'belong' clearly to geology and physics respectively. This is due to the fact that each of them arises out of a discussion which is characteristic of the tradition of the discipline in question. It arises out of the discussion of some theory, or out of empirical tests bearing upon a theory; and theories, as opposed to subject matter, may constitute a discipline (which might be described as a somewhat loose cluster of theories undergoing a process of challenge, change, and growth). But this does not alter the view that the classification into disciplines is comparatively unimportant, and that we are students, not of disciplines, but of problems.

But are there philosophical problems? The present position of English philosophy, which I shall take as my point of departure, originates, I believe, from the late Professor Ludwig Wittgenstein's

[1] This tendency can be explained by the principle that theoretical explanations are the more satisfactory the better they can be supported by *independent* evidence. (This somewhat cryptic remark cannot, I fear, be amplified in the present context.)

influential doctrine that there are none; that all genuine problems are scientific problems; that the alleged problems of philosophy are pseudo-problems; that the alleged propositions or theories of philosophy are pseudo-propositions or pseudo-theories; that they are not false (if false,[1] their negations would be true propositions or theories) but strictly meaningless combinations of words, no more meaningful than the incoherent babbling of a child who has not yet learned to speak properly.[2]

As a consequence, philosophy cannot contain any theories. Its true nature, according to Wittgenstein, is not that of a theory, but that of an activity. The task of all genuine philosophy is that of unmasking philosophical nonsense, and of teaching people to talk sense.

My plan is to take this doctrine [3] of Wittgenstein's as my starting

[1] It is of particular importance in this connection to realise that Wittgenstein's use of the term 'meaningless' is *not* the usual and somewhat vague one according to which an absurdly false assertion (such as '$2 + 3 = 5{\cdot}427$' or 'I can play Bach on the adding machine') may be called 'meaningless'. He called a statement-like expression 'meaningless' only if it is not a properly constructed statement at all, and therefore neither true nor false. Wittgenstein himself gave the example: 'Socrates is identical'.

[2] Since Wittgenstein described his own *Tractatus* as meaningless (see also the next footnote), he distinguished, at least by implication, between revealing and unimportant nonsense. But this does not affect his main doctrine which I am discussing, the non-existence of philosophical problems. (A discussion of other doctrines of Wittgenstein's can be found in the Notes to my *Open Society*, esp. notes 26, 46, 51, and 52 to ch. 11.)

[3] It is easy to detect at once one flaw in this doctrine: the doctrine, it may be said, is itself a philosophic theory, claiming to be true, and not to be meaningless. This criticism, however, is a little too cheap. It might be countered in at least two ways. (1) One might say that the doctrine is indeed meaningless *qua* doctrine, but not *qua* activity. (This is the view of Wittgenstein, who said at the end of his *Tractatus Logico-Philosophicus* that whoever understood the book must realise at the end that it was itself meaningless, and must discard it like a ladder, after having used it to reach the desired height.) (2) One might say that the doctrine is not a philosophical but an empirical one, that it states the historical fact that all 'theories' proposed by philosophers are in fact ungrammatical; that they do not, in fact, conform to the rules inherent in those languages in which they appear to be formulated, that this defect turns out to be impossible to remedy; and that every attempt to express them properly has lead to the loss of their philosophic character (and revealed them, for example, as empirical truisms, or as false statements). These two counter arguments rescue, I believe, the threatened consistency of the doctrine, which in this way indeed becomes 'unassailable', as Wittgenstein puts it by the kind of criticism referred to in this note. (See also the next note but one.)

point (section 2). I shall try (in section 3) to explain it; to defend it, to some extent; and to criticise it. And I shall support all this (in sections 4 to 6) by some examples from the history of scientific ideas.

But before proceeding to carry out this plan, I wish to reaffirm my conviction that a philosopher should philosophise, that is, try to solve philosophic problems, rather than talk about philosophy. If Wittgenstein's doctrine is true, then nobody can, in this sense, philosophise. If this were my opinion, I would give up philosophy. But it so happens that I am not only deeply interested in certain philosophical problems (I do not much care whether they are 'rightly' called 'philosophical problems'), but possessed by the belief that I may even contribute—if only a little, and only by hard work—to their solution. And my only excuse for talking here about philosophy—instead of philosophising—is, in the last resort, my hope that, in carrying out my programme for this address, an opportunity will offer itself of doing a little philosophising, after all.

2

Ever since the rise of Hegelianism there has existed a dangerous gulf between science and philosophy. Philosophers were accused—rightly, I believe—of 'philosophising without knowledge of fact', and their philosophies were described as 'mere fancies, even imbecile fancies'.[1] Although Hegelianism was the leading influence in England and on the Continent, opposition to it, and contempt of its pretentiousness, never died out completely. Its downfall was brought about by a philosopher who, like Leibniz, Kant, and J. S. Mill before him, had a sound knowledge of science, especially mathematics. I am speaking of Bertrand Russell.

Russell is also the author of the classification (closely related to his famous *theory of types*) which is the basis of Wittgenstein's view of philosophy, the classification of the expressions of a language into

(1) *True statements*

(2) *False statements*

(3) *Meaningless expressions*, among which there are statement-like sequences of words, which may be called 'pseudo-statements'.

Russell operated with this distinction in connection with the solution

[1] The two quotations are not the words of a scientific critic, but, ironically enough, Hegel's own characterisation of the philosophy of his friend and forerunner Schelling. Cf. my *Open Society*, note 4 (and text) to ch. 12.

of the logical paradoxes which he discovered. It was essential, for this solution, to distinguish, more especially, between (2) and (3). We might say, in ordinary speech, that a false statement, like ' 3 times 4 equals 173 ' or ' All cats are cows ', is meaningless. Russell, however, reserved this characterisation for expressions such as ' 3 times 4 are cows ' or ' All cats equal 173 ', that is, for expressions which are better not described as false statements (as can easily be seen from the fact that their *prima facie* negations, for example, ' Some cats do not equal 173 ' are no more satisfactory than the original expressions) but as pseudo-statements.

Russell used this distinction mainly for the elimination of the paradoxes (which, he indicated, were meaningless pseudo-statements). Wittgenstein went further. Led, perhaps, by the feeling that what philosophers, especially Hegelian philosophers, were saying was somewhat similar to the paradoxes of logic, he used Russell's distinction in order to denounce all philosophy as meaningless.

As a consequence, there could be no genuine philosophical problems. All alleged philosophical problems could be classified into four classes:[1] (1) those which are purely logical or mathematical, to be answered by logical or mathematical propositions, and therefore not philosophical; (2) those which are factual, to be answered by some statement of the empirical sciences, and therefore again not philosophical; (3) those which are combinations of (1) and (2), and therefore, again, not philosophical; and (4) meaningless pseudo-problems such as ' Do all cows equal 173 ? ' or ' Is Socrates identical ? ' or ' Does an invisible, untouchable, and apparently altogether unknowable Socrates exist ? '

Wittgenstein's idea of eradicating philosophy (and theology) with the help of an adaption of Russell's theory of types was ingenious and original (and more radical even than Comte's positivism which it resembles closely).[2] This idea became the inspiration of the powerful modern school of language analysts who have inherited his belief that there are no genuine philosophical problems, and that all a

[1] Wittgenstein still upheld the doctrine of the non-existence of philosophical problems in the form here described when I saw him last (in 1946, when he presided over a stormy meeting of the Moral Science Club in Cambridge, on the occasion of my reading a paper on ' Are there Philosophical Problems ? '). Since I had never seen any of his unpublished manuscripts which were privately circulated by some of his pupils, I had been wondering whether he had modified what I here call his ' doctrine ' ; but I found his views on this most fundamental and influential point of his teaching unchanged.

[2] Cf. note 52 (2) to ch. 11 of my *Open Society*.

philosopher can do is to unmask and dissolve the linguistic puzzles which have been proposed by traditional philosophy.

My own view of the matter is that only as long as I have genuine philosophical problems to solve shall I continue to take an interest in philosophy. I fail to understand the attraction of a philosophy without problems. I know, of course, that many people talk nonsense; and it is conceivable that it should become one's task (an unpleasant one) to unmask somebody's nonsense, for it may be dangerous nonsense. But I believe that some people have said things which were not very good sense, and certainly not very good grammar, but which are at the same time highly interesting and exciting, and perhaps more worth listening to than the good sense of others. I may mention the differential and integral calculus which, especially in its early forms, was, no doubt, completely paradoxical and nonsensical by Wittgenstein's (and other) standards; which became, however, reasonably well founded as the result of some hundred years of great mathematical efforts; but whose foundations even at this very moment are still in need, and in the process, of clarification.[1] We might remember, in this context, that it was the contrast between the apparent absolute precision of mathematics and the vagueness and inprecision of philosophical language which deeply impressed the earlier followers of Wittgenstein. But had there been a Wittgenstein to use his weapons against the pioneers of the calculus, and had he succeeded in the eradication of their nonsense, where their contemporary critics (such as Berkeley who was, fundamentally, right) failed, then he would have strangled one of the most fascinating and philosophically important developments in the history of thought. Wittgenstein once wrote: 'Whereof one cannot speak, thereof one must be silent.' It was, if I remember rightly, Erwin Schroedinger who replied: 'But it is only here that speaking becomes interesting.' The history of the calculus—and perhaps of his own theory [2]—bears him out.

No doubt, we should all train ourselves to speak as clearly, as

[1] I am alluding to G. Kreisel's recent construction of a monotone bounded sequence of rationals every term of which can be actually computed, but which does not possess a computable limit—in contradiction to what appears to be the *prima facie* interpretation of the classical theorem of Bolzano and Weierstrass, but in agreement with Brouwer's doubts about this theorem. Cf. *Journal of Symbolic Logic*, 1952, **17**, 57.

[2] Before Max Born proposed his famous probability interpretation, Schroedinger's wave equation was, some might contend, meaningless.

precisely, as simply, and as directly as we can. But I believe that there is not a classic of science, or of mathematics, or indeed a book worth reading that could not be shown, by a skilful application of the technique of language analysis, to be full of meaningless pseudo-propositions and what some people might call ' tautologies '.

3

But I have promised to say something in defence of Wittgenstein's views. What I wish to say is, first, that there is much philosophical writing (especially in the Hegelian school) which may justly be criticised as meaningless verbiage ; secondly, that this kind of irresponsible writing was checked, for a time at least, by the influence of Wittgenstein and the language analysts (although it is likely that the most wholesome influence in this respect was the example of Russell who, by the incomparable charm and the clarity of his writings, established the fact that subtlety of content was compatible with lucidity and unpretentiousness of style).

But I am prepared to admit more. In partial defence of Wittgenstein's view, I am prepared to defend the following two theses.

My first thesis is that every philosophy, and especially every philosophical ' school ', is liable to degenerate in such a way that its problems become practically indistinguishable from pseudo-problems, and its cant, accordingly, practically indistinguishable from meaningless babble. This, I shall try to show, is a consequence of philosophical inbreeding. The degeneration of philosophical schools is the consequence of the mistaken belief that one can philosophise without being compelled to turn to philosophy *by problems which arise outside philosophy*—in mathematics, for example, or in cosmology, or in politics, or in religion, or in social life. To put it in other words, my first thesis is this. *Genuine philosophical problems are always rooted in urgent problems outside philosophy, and they die if these roots decay.* In their efforts to solve them, philosophers are liable to pursue what looks like a philosophical method or like a technique or like an unfailing key to philosophical success.[1] But no such methods or techniques exist ; philosophical methods are unimportant, and any method is legitimate

[1] It is very interesting that the imitators were always inclined to believe that the ' master ' did his work with the help of a secret method or a trick. It is reported that in J. S. Bach's days some musicians believed that he possessed a secret formula for the construction of fugue themes.

if it leads to results capable of being rationally discussed. What matters is neither methods nor techniques—nothing but a sensitiveness to problems, and a consuming passion for them ; or as the Greeks said, the gift of wonder.

There are those who feel the urge to solve a problem, those for whom the problem becomes real, like a disorder which they have to get out of their system.[1] They will make a contribution even if they use a method or a technique. But there are others who do not feel this urge, who have no serious and pressing problem but who nevertheless produce exercises in fashionable methods, and for whom philosophy is *application* (of whatever insight or technique you like) rather than *search*. They are luring philosophy into the bog of pseudo-problems and verbal puzzles ; either by offering us pseudo-problems for real ones (the danger which Wittgenstein saw), or by persuading us to concentrate upon the endless and pointless task of unmasking what they rightly or wrongly take for pseudo-problems (the trap into which Wittgenstein fell).

My second thesis is that what appears to be the *prima facie* method of teaching philosophy is liable to produce a philosophy which answers Wittgenstein's description. What I mean by '*prima facie* method of teaching philosophy ', and what would seem to be the only method, is that of giving the beginner (whom we take to be unaware of the history of mathematical, cosmological, and other ideas of science as well as of politics) the works of the great philosophers to read ; say, of Plato and Aristotle, Descartes and Leibniz, Locke, Berkeley, Hume, Kant, and Mill. What is the effect of such a course of reading ? A new world of astonishingly subtle and vast *abstractions* opens itself to the reader, abstractions of an extremely high and difficult level. Thoughts and arguments are put before his mind which sometimes are not only hard to understand, but whose relevance remains obscure since he cannot find out what they may be relevant to. Yet the student knows that these are the great philosophers, that this is the way of philosophy. Thus he will make an effort to adjust his mind to what he believes (mistakenly, as we shall see) to be their way of thinking. He will attempt to speak their queer language, to match the torturous spirals of their argumentation, and perhaps even tie himself up in their curious knots. Some may learn these tricks in a superficial way,

[1] I am alluding to a remark by Professor Gilbert Ryle, who says on page 9 of his *Concept of Mind* : 'Primarily I am trying to get some disorders out of my own system.'

others may begin to become genuinely fascinated addicts. Yet I feel that we ought to respect the man who, having made his effort, comes ultimately to what may be described as Wittgenstein's conclusion : 'I have learned the jargon as well as anybody. It is very clever and captivating. In fact, it is dangerously captivating ; for the simple truth about the matter is that it is much ado about nothing —just a lot of nonsense.'

Now I believe such a conclusion to be grossly mistaken ; it is, however, the almost inescapable result, I contend, of the *prima facie* method of teaching philosophy here described. (I do not deny, of course, that some particularly gifted students may find very much more in the works of the great philosophers than this story indicates—and without deceiving themselves.) For the chance of finding out the extra-philosophical problems (the mathematical, scientific, moral and political problems) which inspired these great philosophers is very small indeed. These problems can be discovered, as a rule, only by studying the history of, for example, scientific ideas, and especially the problem-situation in mathematics and the sciences of the period in question ; and this, in turn, presupposes a considerable acquaintance with mathematics and science. Only an understanding of the contemporary problem-situation in the sciences can enable the student of the great philosophers to understand that they tried to solve urgent and concrete problems ; problems which, they found, could not be dismissed. And only after understanding this fact can a student attain a different picture of the great philosophies—one which makes full sense of the apparent nonsense.

I shall try to establish my two theses with the help of examples ; but before turning to these examples, I wish to summarise my theses, and to balance my account with Wittgenstein.

My two theses amount to the contention that philosophy is deeply rooted in non-philosophical problems ; that Wittengstein's negative judgment is correct, by and large, as far as philosophies are concerned which have forgotten their extra-philosophical roots ; and that these roots are easily forgotten by philosophers who 'study' philosophy, instead of being forced into philosophy by the pressure of non-philosophical problems.

My view of Wittgenstein's doctrine may be summed up as follows. It is true, by and large, that pure philosophical problems do not exist ; for indeed, the purer a philosophical problem becomes, the more is lost of its original sense, significance, or meaning, and the more liable is its discussion to degenerate into empty verbalism. On the other hand, there exist not only genuine scientific problems, but genuine philosophical problems. Even if, upon analysis, these problems turn out to have factual components, they need not be classified as belonging to science. And even if they should be soluble by, say, purely logical means, they need not be classified as purely logical or tautological. Analogous situations arise in physics. For example, the explanation or prediction of certain spectral terms (with the help of a hypothesis concerning the structure of atoms) may turn out to be soluble by purely mathematical calculations. But this, again, does not imply that the problem belonged to pure mathematics rather than to physics. We are perfectly justified in calling a problem 'physical' if it is connected with problems and theories which have been traditionally discussed by physicists (such as the problems of the constitution of matter), even if the means used for its solution turn out to be purely mathematical. As we have seen, the solution of problems may cut through the boundary of many sciences. Similarly, a problem may be rightly called 'philosophical' if we find that, although originally it may have arisen in connection with, say, atomic theory, it is more closely connected with the problems and theories which have been discussed by philosophers than with theories nowadays treated by physicists. And again, it does not matter in the least what kind of methods we use in solving such a problem. Cosmology, for example, will always be of great philosophical interest, even though by some of its methods it has become closely allied to what is perhaps better called 'physics'. To say that, since it deals with factual issues, it must belong to science rather than to philosophy, is not only pedantic but clearly the result of an epistemological, and thus of a philosophical, dogma. Similarly, there is no reason why a problem soluble by logical means should be denied the attribute 'philosophical'. It may well be typically philosophical, or physical, or biological. For example, logical analysis played a considerable part in Einstein's special theory of relativity; and it was, partly, this fact which made this theory philosophically interesting, and which gave rise to a wide range of philosophical problems connected with it.

Wittgenstein's doctrine turns out to be the result of the thesis that all genuine statements (and therefore all genuine problems) can be classified into one of two exclusive classes: factual statements (*synthetic a posteriori*), and logical statements (*analytic a priori*). This simple dichotomy, although extremely valuable for the purposes of a rough survey, turns out to be for many purposes too simple.[1] But although it is, as it were, specially designed to exclude the existence of philosophical problems, it is very far from achieving even this aim ; for even if we accept the dichotomy, we can still claim that factual or logical or mixed problems may turn out, in certain circumstances, to be philosophical.

4

I now turn to my first example : *Plato and the Crisis in Early Greek Atomism.*

My thesis here is that Plato's central philosophical doctrine, the so-called Theory of Forms or Ideas, cannot be properly understood except in an extra-philosophical context[2] ; more expecially in the context of the critical problem situation in Greek science[3] (mainly

[1] Already in my *Logic der Forschung* (Vienna, 1935), I pointed out that a theory such as Newton's may be *interpreted* either as factual or as consisting of implicit definitions (in the sense of Poincaré and Eddington), and that the interpretation which a physicist adopts exhibits itself in his *attitude* towards tests which go against his theory rather than in what he says. The dogma of the simple dichotomy has been recently attacked, on very different lines, by F. H. Heinemann (*Proc. of the Xth Intern. Congress of Philosophy* (Amsterdam, 1949), Fasc. **2**, 629, Amsterdam, 1949), by W. van O. Quine, and by Morton G. White. It may be remarked, again from a different point of view, that the dichotomy applies, in a precise sense, only to a formalised language, and therefore is liable to break down for those languages in which we must speak prior to any formalisation, i.e. in those languages in which all the traditional problems were conceived. Some members of the school of the language analysts, however, still believe it a sound method to unmask a theory as 'tautological'.

[2] In my *Open Society and Its Enemies*, I have tried to explain in some detail another extra-philosophical root of the same doctrine, viz. a political root. I also discussed there (in note 9 to ch. 6 of the revised 4th edition, 1952) the problem with which I am concerned in the present section, but from a somewhat different angle. The note referred to and the present section partly overlap ; but they are largely supplementary to each other. Relevant references (esp. to Plato) omitted here will be found there.

[3] There are historians who deny that the term 'science' can be properly applied to any development which is older than the sixteenth or even the seventeenth century. But quite apart from the fact that controversies about labels should be avoided, there can, I believe, no longer be a doubt nowadays about the astonishing similarity, not to say identity, of the aims, interests, activities, arguments, and methods, of, say, Galileo and Archimedes, or Copernicus and Plato, or Kepler and Aristarchus (the 'Copernicus of antiquity'). And any doubt concerning the extreme age of scientific

in the theory of matter) which developed as a consequence of *the discovery of the irrationality of the square root of two*. If my thesis is correct, then Plato's theory has not so far been fully understood. (Whether a 'full' understanding can ever be achieved is, of course, most questionable.) But the more important consequence would be that it can never be understood by philosophers trained in accordance with the *prima facie* method described in the foregoing section—unless, of course, they are specially and *ad hoc* informed of the relevant facts (which they may have to accept on authority).

It is well known [1] that Plato's Theory of Forms is historically as well as in its content closely connected with the Pythagorean theory that all things are, in essence, numbers. The details of this connection, and the connection between Atomism and Pythagoreanism, are perhaps not so well known. I shall therefore tell the whole story in brief, as I see it at present.

It appears that the founder of the Pythagorean order or sect was deeply impressed by two discoveries. The first discovery was that a *prima facie* purely qualitative phenomenon such as musical harmony was, in essence, based upon the purely numerical ratios 1 : 2 ; 2 : 3 ; 3 : 4. The second was that the 'right' or 'straight' angle (obtainable for example by folding a leaf twice, so that the two folds form a cross) was connected with the purely numerical ratios 3 : 4 : 5, or 5 : 12 : 13 (the sides of rectangular triangles). These two discoveries, it appears, led Pythagoras to the somewhat fantastic generalisation that all things are, in essence, numbers, or ratios of numbers ; or that number was the ratio (*logos* = reason), the rational essence of things, or their real nature.

Fantastic as this idea was, it proved in many ways fruitful. One of its most successful applications was to simple geometrical figures,

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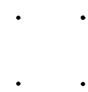
observation, and of careful computations based upon observation, has been dispelled nowadays by the discovery of new evidence concerning the history of ancient astronomy. We can now draw not only a parallel between Tycho and Hipparchus, but even one between Hansen (1857) and Cidenas the Chaldean (314 B.C.), whose computations of the 'constants for the motion of Sun and Moon' are without exception comparable in precision to those of the best nineteenth-century astronomers, 'Cidenas' value for the motion of the Sun from the Node (0″·5 to great), although inferior to Brown's, is superior to at least one of the most widely used modern values', wrote J. K. Fotheringham in 1928, in his most admirable article 'The Indebtedness of Greek to Chaldean Astronomy' (*The Observatory*, 1928, **51**, No. 653), upon which my contention concerning the age of astronomy is based.

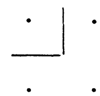
[1] From Aristotle's *Metaphysics*

such as squares, rectangular and isosceles triangles, and also to certain simple solids, such as pyramids. The treatment of some of these geometrical problems was based upon the so-called *gnōmōn*.

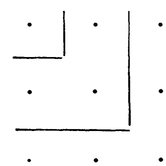
This can be explained as follows. If we indicate a square by four dots,



we may interpret this as the result of adding three dots to the one dot on the upper left corner. These three dots are the first *gnōmōn* ; we may indicate it thus :

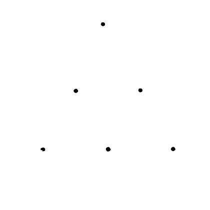


By adding a second *gnōmōn*, consisting of five more dots, we obtain

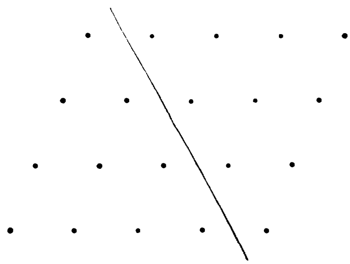


One sees at once that every number of the sequence of the odd numbers, 1, 3, 5, 7 . . . , each forms a *gnōmōn* of a square, and that the sums 1, 1 + 3, 1 + 3 + 5, 1 + 3 + 5 + 7, . . . are the square numbers, and that, if n is the (number of dots in the) side of a square, its area (total number of dots = n^2) will be equal to the sum of the first n odd numbers.

As with the treatment of squares, so with the treatment of isosceles triangles.



Here each *gnōmōn* is a last horizontal line of points, and each element of the sequence 1, 2, 3, 4, . . . is a *gnōmōn*. The ' triangular numbers ' are the sums 1 + 2 ; 1 + 2 + 3 ; 1 + 2 + 3 + 4, etc., that is, the sums of the first n natural numbers. By putting two such triangles side by side



we obtain the parallelogram with the horizontal side $n + 1$ and the other side n, containing $n(n + 1)$ dots. Since it consists of two isosceles triangles, its number is $2(1 + 2 + \ldots + n)$, so that we obtain the equation

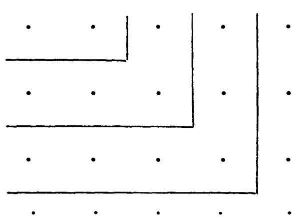
(3) $$1 + 2 + \ldots + n = \tfrac{1}{2}n(n + 1)$$

and

(4) $$d(1 + 2 + \ldots + n) = \frac{d}{2}n(n + 1).$$

From this it is easy to obtain the general formula for the sum of an arithmetical series.

We also obtain ' oblong numbers ', that is the numbers of oblong rectangular figures, of which the simplest is



with the oblong numbers $2 + 4 + 6 \ldots$, i.e. the *gnōmōn* of an oblong is an even number, and the oblong numbers are the sums of the even number.

These considerations were extended to solids; for example, by summing the first triangular number, pyramid numbers were obtained. But the main application was to plain figures, or shapes, or ' Forms '. These, it was believed, are characterised by the appropriate sequence of numbers, and thus by the numerical ratios of the consecutive numbers of the sequence. In other words, *' Forms' are numbers or ratios of numbers.* On the other hand, not only shapes of things, but also abstract properties, such as harmony, and ' straightness ' are numbers. In this way, the general theory that numbers are the rational essences of all things, is arrived at with some plausibility.

It is very probable that the development of this view was influenced by the similarity of the dot-diagrams with the diagram of a constellation such as the Lion, or the Scorpion, or the Virgo. If a Lion is an arrangement of dots, it must have a number. In this way the belief seems to have arisen that the numbers, or ' Forms ', are heavenly shapes of things.

One of the main elements of this early theory was the so-called ' Table of Opposites ', based upon the fundamental distinction between odd and even numbers. It contains such things as

ONE	MANY
ODD	EVEN
MALE	FEMALE
REST (BEING)	CHANGE (BECOMING)
DETERMINATE	INDETERMINATE
SQUARE	OBLONG
STRAIGHT	CROOKED
RIGHT	LEFT
LIGHT	DARKNESS
GOOD	BAD

In reading through this strange table one gets some idea of the working of the Pythagorean mind, and why not only the ' Forms ' or shapes of geometrical figures were considered to be numbers, in essence, but also abstract ideas, such as Justice and, of course, Harmony, and Health, Beauty and Knowledge. The table is interesting also because it was taken over, with very little alteration, by Plato. Plato's famous theory of ' Forms ' or ' Ideas ' may indeed be described, somewhat roughly, as the doctrine that the ' Good ' side of the Table of Opposites constitutes an (invisible) Universe, a Universe of Higher Reality, of the Unchanging and Determinate ' Forms ' of all things, and that True and Certain Knowledge (*epistēmē* = *scientia* = *science*) can be of this Unchanging and Real Universe only, while the visible world of change and flux in which we live and die, the world of generation and destruction, the world of experience, is only a kind of reflection or copy of that Real World. It is only a world of appearance, of which no True and Certain Knowledge can be obtained. What can be obtained in the place of Knowledge (*epistēmē*) are only the plausible but uncertain and prejudiced opinions (*doxa*) of fallible mortals.[1] In his interpretation of the Table of Opposites, Plato was influenced by Parmenides, the man who stimulated the development of Democritus' atomic theory.

Returning now to the original Pythagorean view, there is one thing in it which is of decisive importance for our story. It will have been observed that the Pythagorean emphasis upon Number was fruitful from the point of view of the development of scientific ideas. This is often but somewhat loosely expressed by saying that the Pythagoreans encouraged numerical scientific measurements. Now the point which we must realise is that, for the Pythagoreans, all this was *counting rather than measuring*. It was the counting of numbers, of invisible essences or ' Natures ' which were Numbers of little dots or stigmata. Admittedly, we cannot count these little dots directly, since they are invisible. What we actually do is not to count the Numbers or Natural Units, but to measure, i.e. to count arbitrary visible units. But the significance of measurements was interpreted as revealing, indirectly, the true *Ratios of the Natural Units or of the Natural Numbers.*

Thus Euclid's methods of proving the so-called ' Theorem of Pythagoras ' (Euclid's I, 47) according to which, if a is the side of a triangle opposite to its right angle between b and c,

$$a^2 = b^2 + c^2, \tag{1}$$

was completely foreign to the spirit of Pythagorean mathematics. In spite of the fact that the theorem was known to the Babylonians and geometrically proved by them, neither Pythagoras nor Plato appear to have known the general *geometrical* proof; for the problem for which they offered solutions, the *arithmetical* one of finding the integral solutions for the sides of rectangular triangles, can be easily

[1] Plato's distinction (*epistēmē* vs. *doxa*) derives, I think, from Parmenides (*truth* vs. *seeming*). Plato clearly realised that *all* knowledge of the visible world, the changing world of appearance, consists of *doxa*; that it is tainted by uncertainty even if it utilises the *epistēmē*, the knowledge of the unchanging ' Forms ' and of pure mathematics, to the utmost, and even if it interprets the visible world with the help of a theory of the invisible world. Cf. *Cratylus*, 439b ff., *Rep.* 476d ff.; and especially *Timaeus*, 29b ff., where the distinction is applied to those parts of Plato's own theory which we should nowadays call ' physics ' or ' cosmology ', or, more generally, ' natural science '. They belong, Plato says, to the realm of *doxa* (in spite of the fact that science = *scientia* = *epistēmē*; cf. my remarks on this problem in *The Philosophical Quarterly*, April 1952, p. 168). For a different view concerning Plato's relation to Parmenides, see Sir David Ross, *Plato's Theory of Ideas*, Oxford, 1951, p. 164.

solved, if (1) is known by the formula (m and n are natural numbers, and $m > n$)

$$a = m^2 + n^2 \; ; \; b = 2mn \; ; \; c = m^2 - n^2. \tag{2}$$

But formula (2) was unknown to Pythagoras and even to Plato. This emerges from the tradition according to which Pythagoras proposed the formula

$$a = 2m(m + 1) + 1 \; ; \; b = 2m(m + 1) \; ; \; c = 2m + 1 \tag{3}$$

which can be read off the *gnōmōn* of the square numbers, but which is less general than (2), since it fails, for example, for 8 : 15 : 17. To Plato, who is reported to have improved Pythagoras' formula (3), is attributed another formula which still falls short of the general solution (2).

We now come to the discovery of the *irrationality of the square root of two*. According to tradition, this discovery was made within the Pythagorean order, but was kept secret. (This is suggested by the old term for 'irrational', '*arrhētos*', that is, 'unspeakable', which might well have meant 'the unspeakable mystery'.) This discovery struck at the root of Pythagoreanism; for it meant that such a simple geometrical entity as the diagonal d of the square with the side a could demonstrably not be characterised by *any* ratio of natural numbers; $d : a$ was *no ratio*. The tradition has it that the member of the school who gave away the secret was killed for his treachery. However this may be, there is little doubt that the realisation of the fact that irrational magnitudes (they were, of course, not recognised as numbers) existed, and that their existence could be proved, led to the downfall of the Pythagorean order.

The Pythagorean theory, with its dot-diagrams, contains, no doubt, the suggestion of a very primitive atomism. How far the atomic theory of Democritus was influenced by Pythagoreanism is difficult to assess. Its main influences came, one can say for certain, from the Eleatic School: from Parmenides and from Zeno. The basic problem of this school, and of Democritus, was that of the rational understanding of *change*. (I differ here from the interpretations of Cornford and others.) I think that this problem derives from Ionian rather than from Pythagorean thought, and that it has remained the fundamental problem of Natural Philosophy.

Although Parmenides himself was not a physicist (as opposed to his great Ionian predecessors), he may be described, I believe, as having fathered *theoretical physics*. He produced an anti-physical theory which, however, was the first hypothetical-deductive system. And it was the beginning of a long series of such systems of physical theories each of which was an improvement on its predecessor. As a rule the improvement was found necessary by the realisation that the earlier system was falsified by certain facts of experience. Such an empirical refutation of the consequences of a deductive system leads to efforts at its reconstruction, and thus to a new and improved theory which, as a rule, clearly bears the mark of its ancestry, of the older theory as well as of the refuting experience.

These experiences or observations were, we shall see, very crude at first, but they became more and more subtle as the theories became more and more capable of accounting for the cruder observations. In the case of Parmenides' theory, the clash with observation was so obvious that it would seem perhaps fanciful to describe the theory as the first hypothetical-deductive system of physics. We may, therefore, describe it as the last pre-physical deductive system, whose falsification gave rise to the first truly physical theory, the atomistic theory of Democritus.

Parmenides' theory is simple. He finds it impossible to understand change or movement rationally, and concludes that there is really no change—or that change is only apparent. But before we indulge in feelings of superiority, in the face of such a hopelessly unrealistic theory, we should first realise that there is a serious problem here. If a thing X changes, then clearly it is no longer the same thing X. On the other hand, we cannot say that X changes without implying that X persists during the change ; that it is the same thing X, at the beginning and at the end of the change. Thus, it appears that we arrive at a contradiction, and that the idea of a thing that changes, and therefore the idea of change, is impossible.

All this sounds very philosophical and abstract, and so it is. But it is a fact that the difficulty here indicated has never ceased to make itself felt in the development of physics.[1] And a deterministic system such as that of Einstein's field theory might even be described as a four-dimensional version of Parmenides' unchanging three-dimensional universe. For, in a sense, no change occurs in Einstein's four-dimensional block-universe. Everything is there just as it is, in its four-dimensional *locus* ; change becomes a kind of ' apparent ' change ; it is ' only ' the observer who, as it were, glides along his

[1] This may be seen from Emile Meyerson's *Identity and Reality*, one of the most interesting philosophical studies of the development of physical theories.

world-line and becomes successively conscious of the different *loci* along this world-line, that is, of his spatio-temporal surrounding. . . .

To ıeturn from this new Parmenides to the older father of theoretical physics, we may present his deductive theory roughly as follows.

(1) Only what is, is.
(2) What is not does not exist.
(3) Non-being, that is, the void, does not exist.
(4) The world is full.
(5) The world has no parts; it is one huge block (because it is full).
(6) Motion is impossible (since there is no empty space into which anything could move).

The conclusions (5) and (6) were obviously contradicted by facts. Thus Democritus argued from the falsity of the conclusion to that of the premises:

(6′) There is motion (thus motion is possible).
(5′) The world has parts; it is not one, but many.
(4′) Thus the world cannot be full.[1]
(3′) The void or (non-being) exist.

So far the theory had to be altered. With regard to being, or to the many existing things (as opposed to the void), Democritus adopted Parmenides' theory that they had no parts. They were indivisible (atoms), because they were full, because they had no void inside.

The central point of this theory is that it gives a rational account of change. The world consists of empty space (the void) with atoms in it. The atoms do not change; they are Parmenidean indivisible block universes in miniature.[2] All change is due to rearrangement of atoms in space. Accordingly, *all change is movement.* Since the only kind of novelty possible is novelty of arrangement, it is, in principle, possible *to predict all future changes in the world*, provided we manage to predict the motion of mass-points.

[1] The inference from the existence of motion to that of a void does not follow, because Parmenides' inference from the fullness of the world to the impossibility of motion does not follow. Plato seems to have been the first to see, if only dimly, that in a full world circular or vortex-like motion is possible, provided that there is a liquid-like medium in the world. (Peas can move with the vortices of pea-soup.) This idea, first offered somewhat half-heartedly in the *Timaeus*, becomes the basis of Cartesianism and of the light-ether theory as it was held down to 1905.

[2] Democritus' theory admitted also large block-atoms, but the vast majority of his atoms were invisibly small.

Democritus' theory of change was of tremendous importance for the development of physical science. It was partly accepted by Plato, who retained much of atomism but explained change not only by unchanging yet moving atoms, but also by the ' Forms ' which were subject neither to change nor to motion. But it was condemned by Aristotle who taught in its stead that all change was the unfolding of the inherent potentialities of essentially unchanging *substances*. But although Aristotle's theory of substances as the subjects of change became dominant, it proved barren ; [1] and Democritus' theory that all change must be explained by movement became the tacitly accepted official programme of physics down to our own day. It is still part of the philosophy of physics, in spite of the fact that physics itself has outgrown it (to say nothing of the biological and social sciences). For with Newton, in addition to moving mass-points, *forces* of changing intensity (and direction) enter the scene. True, these changes can be explained as due to, or dependent upon, motion, that is upon the changing position of particles, but they are nevertheless not identical with the changes in position ; owing to the square law, the dependence is not even a linear one. And with Faraday and Maxwell, changing fields of forces become as important as material atomic particles. That our modern atoms turn out to be composite is a minor matter ; from Democritus' point of view, not our atoms but rather our elementary particles would be real atoms—except that these too turn out to be liable to change. Thus we have a most interesting situation. A philosophy of change, designed to meet the difficulty of understanding change rationally, serves sciences for thousands of years, but is ultimately superseded by the development of science itself ; and this fact passes practically unnoticed by philosophers who are busily denying the existence of philosophical problems.

Democritus' theory was a marvellous achievement. It provided a theoretical framework for the explanation of most of the empirically known properties of matter (discussed already by the Ionians), such as compressibility, degrees of hardness and resilience, rarefaction and condensation, coherence, disintegration, combustion, and many others.

[1] The barrenness of the ' essentialist ' (cf. note 2 above) theory of substance is connected with its anthropomorphism ; for substances (as Locke saw) take their plausibility from the experience of a self-identical but changing and unfolding self. But although we may welcome the fact that Aristotle's substances have disappeared from *physics*, there is nothing wrong, as Professor Hayek says, in thinking anthropomorphically about *man* ; and there is no reason why they should disappear from psychology.

But apart from being important as an explanation of the phenomena of experience, the theory was important in other ways. First, it established the methodological principle that a deductive theory or explanation must ' save the phenomena ', that is, must be in agreement with experience. Secondly, it showed that a theory may be speculative, and based upon the fundamental (Parmenidean) principle that the world as it must be understood by argumentative thought turns out to be different from the world of *prima facie* experience, from the world as seen, heard, smelled, tasted, touched ; [1] and that such a speculative theory may nevertheless accept the empiricist ' criterion ' that it is the visible that decides the acceptance or rejection of a theory of the invisible [2] (such as the atoms). This philosophy has remained fundamental to the whole development of physics, and has continued to conflict with all ' relativistic ' [3] and ' positivistic ' [4] tendencies.

Furthermore, Democritus' theory led to the first successes of the method of exhaustion (the forerunner of the calculus of integration), since Archimedes himself acknowledged that Democritus was the first to formulate the theory of the volumes of cones and pyramids.[5] But perhaps the most fascinating element in Democritus' theory is his doctrine of the quantisation of space and time. I have in mind the doctrine, now extensively discussed,[6] that there is a shortest distance and a smallest time interval, that is to say, distances in space and time (elements of length and time, Democritus' *ameres* [7] in contradistinction to his atoms) such that no smaller ones are measurable.

[1] Cf. Democritus, Diels, fragm. 11 (cf. Anaxagoras, Dield fragm. 21 ; see also fragm. 7).

[2] Cf. Sextus Empiricus, *Adv. mathem.* (Bekker) vii. 140, p. 221, 23B.

[3] ' Relativistic ' in the sense of philosophical relativism, e.g. of Protagoras' *homo mensura* doctrine. It is, unfortunately, still necessary to emphasise that Einstein's theory has nothing in common with philosophical relativism.

[4] Such as those of Bacon ; the theory (but fortunately not the practice) of the early *Royal Society* ; and in our time, of Mach (who opposed atomic theory) ; and of the sense-data theorists.

[5] Cf. Diels, fragm. 155, which must be interpreted in the light of Archimedes (ed. Heiberg) II^2, p. 428 f. Cf. S. Luria's most important article ' Die Infinitesimalmethode der antiken Atomisten ' (*Quellen & Studien zur Gesch. d. Math.* Abt. B. Bd. 2, Heft 2 (1932), p. 142).

[6] Cf. A. March, *Natur und Erkenntnis*, Vienna, 1948, p. 193 f.

[7] Cf. S. Luria, *op. cit.*, esp. pp. 148 ff., 172 ff.

5

Democritus' atomism was developed and expounded as a point for point reply [1] to the detailed arguments of his Eleatic predecessors, of Parmenides and of his pupil, Zeno. Especially Democritus' theory of atomic distances and time intervals is the direct result of Zeno's arguments, or more precisely, of the rejection of Zeno's conclusions. But nowhere in Zeno is there an allusion to the discovery of irrationals.

We do not know the date of the proof of the irrationality of the square root of two, or the date when the discovery became publicly known. Although there existed a tradition ascribing it to Pythagoras (sixth century B.C.), and although some authors [2] call it the 'theorem of Pythagoras', there can be little doubt that the discovery was not made, and certainly not publicly known, before 450 B.C., and probably not before 420. Whether Democritus knew about it is uncertain. I now feel inclined to believe that he did not; and that the title of Democritus' two lost books, *Peri alogōn grammōn kai kastōn*, should be translated *On Illogical Lines and Full Bodies* (*Atoms*),[3] and that

[1] This point for point reply is preserved in Aristotle's *On Generation and Corruption*, 316a 14 ff., a very important passage first identified as Democritean by I. Hammer Jensen in 1910 and carefully discussed by Luria who says (*op. cit.* 135) of the Parmenides and Zeno: 'Democritus borrows their deductive arguments, but he arrives at the opposite conclusion.'

[2] Cf. G. H. Hardy and H. M. Wright, *Introduction to the Theory of Numbers* (1938), pp. 39, 42, where a very interesting historical remark on Theodorus' proof, as reported in Plato's *Theaeteteus*, will be found.

[3] Rather than *On Irrational Lines and Atoms*, as I translated in note 9 to ch. 6 of my *Open Society* (revised ed.). What is probably meant by the title (considering Plato's passage mentioned in the next note) might, I think, be best rendered by '*On Crazy Distances and Atoms*'. Cf. H. Vogt, *Bibl. Math.*, 1910, 10, 147, and S. Luria, *op. cit.* pp. 168 ff., where it is convincingly suggested that (Arist.) *De insec. lin.* 968b 17 and Plutarch, *De comm. notit.,* 38, 2, p. 1078 f., contain traces of Democritus' work. According to these sources, Democritus' argument was this. *If lines are infinitely divisible*, then they are composed of an infinity of ultimate units and are therefore *all* related like $\infty : \infty$, that is to say, they are all 'non-comparable' (there is no proportion). Indeed, if lines are considered as classes of points, the 'number' (potency) of the points of a line is, according to modern views, equal for all lines, whether the lines are finite or infinite. This fact has been described as 'paradoxical' (for example, by Bolzano) and might well have been described as 'crazy' by Democritus. It may be noted that, according to Brouwer, even the classical theory of the *measure* of a continuum leads to fundamentally the same results; since he asserts that all classical continua have zero measure, the absence of a ratio is here expressed by $0 : 0$. Democritus' result (and his theory of *ameres*) appears to be inescapable as long as geometry is based on the Pythagorean *arithmetical method*, i.e. on the counting of dots.

these two books do not contain any reference to the problem of irrationality.[1]

My belief that Democritus did not know about irrationalities is based on the fact that there are no traces of a defence of his theory against the fatal blow which it received from this discovery. For the blow was as fatal to Atomism as it was to Pythagoreanism. Both theories were based on the doctrine that all measurement is, ultimately, counting of natural units, so that every measurement must be reducible to pure numbers. The distance between any two atomic points must, therefore, consist of a certain number of atomic distances; thus all distances must be commensurable. But this turns out to be impossible even in the simple case of the distances between the corners of a square, because of the incommensurability of its diagonal with its side.

It was Plato who realised this fact, and who in the *Laws* stressed its importance in the strongest possible terms, denouncing his compatriots for their failure to realise what it meant. It is my contention that his whole philosophy, and especially his theory of 'Forms' or 'Ideas', was influenced by it.

Plato was very close to the Pythagoreans as well as to the Eleatic Schools; and although he appears to have felt antipathetic to Democritus, he was himself an atomist of a kind. (Atomist teaching remained as one of the school traditions of the Academy.[2]) This is not surprising in view of the close relation between Pythagorean and atomistic ideas. But all this was threatened by the discovery of the irrational. I suggest that Plato's main contribution to science sprang from his realisation of the problem of the irrational, and from the modification of Pythagoreanism and atomism which he undertook in order to rescue science from a catastrophic situation.

He realised that the purely arithmetical theory of nature was defeated, and that a new mathematical method for description and explanation of the world was needed. Thus he encouraged the development of an autonomous geometrical method which found its fulfilment in the 'Elements' of the Platonist Euclid.

What are the facts? I shall try to put them all briefly together.

(1) Pythagoreanism and atomism in Democritus' form were both fundamentally based on arithmetic, that is on counting.

[1] This would be in keeping with the fact mentioned in the note cited from the *Open Society*, that the term '*alogos*' is only much later known to be used for 'irrational', and that Plato who (*Repub.* 534d) alludes to Democritus' title, nevertheless never uses '*alogos*' as a synonym for '*arrhētos*'.

[2] See S. Luria, esp. on Plutarch, *loc. cit.*

(2) Plato emphasised the catastrophic character of the discovery of the irrationals.

(3) He inscribed over the door of the Academy : ' Nobody untrained in geometry may enter my house '. But *geometry*, according to Plato's immediate pupil Aristotle as well as to Euclid, treats of incommensurables or irrationals, in contradistinction to *arithmetic* which treats of ' the odd and the even '.

(4) Within a short time after Plato's death, his school produced, in Euclid's *Elements*, a work whose main point was that it freed mathematics from the ' arithmetical ' assumption of commensurability or rationality.

(5) Plato himself contributed to this development, and especially to the development of solid geometry.

(6) More especially, he gave in the *Timaeus* a specifically geometrical version of the formerly purely arithmetical atomic theory, that is, a version which constructed the elementary particles (the famous Platonic bodies) out of triangles which incorporated the irrational square roots of two and of three. (See below.) In most other respects, he preserved both Pythagorean ideas as well as some of the most important ideas of Democritus.[1] At the same time, he tried to eliminate Democritus' void ; for he realised that motion remains possible even in a ' full ' world, provided motion is conceived as of the character of vortices in a liquid. Thus he retained some of the most fundamental ideas of Parmenides.[2]

(7) Plato encouraged the construction of geometrical models of the world, and especially models explaining the planetary movements. Euclid's geometry was not intended as an exercise in pure geometry (as now usually assumed), but as a *theory of the world*. Ever since [3]

[1] Plato took over, more especially, Democritus' theory of vortices (Diels, fragm. 167, 164 ; cf. Anaxagoras, Diels 9 ; and 12, 13) ; see also the next footnote, and his theory of what we nowadays would call gravitational phenomena (Diels, 164 ; Anaxagoras, 12, 13, 15, and 2)—a theory which, slightly modified by Aristotle, was ultimately discarded by Galileo.

[2] Plato's reconciliation of atomism and the theory of the *plenum* (' nature abhors the void ') became of the greatest importance for the history of physics down to our own day. For it influenced Descartes strongly, became the basis of the theory of ether and light, and thus ultimately, via Huyghens and Maxwell, of de Broglie's and of Schroedinger's wave mechanics.

[3] The only exception is the partial reappearance of arithmetical methods in the New Quantum Theory, e.g. in the electron shell theory of the periodic system based upon Pauli's exclusion principle.

Plato and Euclid, but not before, it has been taken for granted that geometry (rather than arithmetic) is the fundamental instrument of all physical explanations and descriptions, of the theory of matter as well as of cosmology.[1]

These are the historical facts. They go a long way, I believe, to establish my contention that what I have described as the *prima facie* method of philosophy cannot lead to an understanding of the problems which inspired Plato. Nor can it lead to an appreciation of what may be justly claimed to be his greatest philosophical achievement, the geometrical theory of the world which became the basis of the works of Euclid, Aristarchus, Archimedes, Copernicus, Kepler, Galileo, Descartes, Newton, Maxwell, and Einstein.

But is this achievement properly described as philosophical ? Does it not rather belong to physics—a factual science—and to pure mathematics—a branch, Wittgenstein's school would contend, of tautological logic ?

I believe that we can at this stage see fairly clearly why Plato's achievement (although it has no doubt its physical, its logical, its mixed, and its nonsensical components) was a philosophical achievement ; why at least part of his philosophy of nature and of physics has lasted and, I believe, will last.

What we find in Plato and his predecessors is the conscious construction and invention of a new approach towards the world and towards the knowledge of the world. This approach transforms a fundamentally theological idea, *the idea of explaining the visible world by a postulated invisible world*,[2] into the fundamental instrument of theoretical science. The idea was explicitly formulated by Anaxagoras and Democritus [3] as the principle of investigations into the nature of matter or body ; visible matter was to be explained by hypotheses

Concerning the modern tendency towards what is sometimes called ' arithmetisation of geometry ' (a tendency which is hardly characteristic of all modern work on geometry), it should be noted that there is little similarity with the Pythagorean approach since *infinite sequences of* natural numbers are its main instrument rather than the natural numbers themselves.

[1] For a similar view of Plato's and Euclid's influence, see G. F. Hemens, *Proc. of the Xth Intern. Congress of Philosophy* (Amsterdam 1949), Fasc. 2, 847.

[2] Cf. Homer's explanation of the visible world before Troy with the help of the invisible world of the Olympus. The idea loses, with Democritus, some of its theological character (which is still strong in Parmenides, although less so in Anaxagoras) but regains it with Plato, only to lose it soon afterwards.

[3] See the references given above.

about invisibles, about an *invisible structure which is too small to be seen*. With Plato, this ideas is consciously accepted and generalised ; the visible world of change is ultimately to be explained by an invisible world of unchanging ' Forms ' (or substances or essences, or ' natures ' —as we shall see, geometrical shapes or figures).

Is this idea about the invisible structure of matter a physical or a philosophical idea ? If a physicist *acts* upon this theory, that is to say, if he accepts it, perhaps even without becoming conscious of it, by accepting the traditional problems of his subject, as presented by the problem-situation with which he is confronted ; and if he, so acting, produces a new specific theory of the structure of matter ; then I should not call him a philosopher. But if he reflects upon it, and, for example, rejects it (like Berkeley or Mach), preferring a phenomenological or positivistic physics to the theoretical and somewhat theological approach, then he may be called a philosopher. Similarly, those who consciously searched for the theoretical approach, who constructed it, and who explicitly formulated it, and thus transferred the hypothetical-deductive method from the field of theology to that of physics, were philosophers, even though they were physicists in so far as they acted upon their own precepts and tried to produce actual theories of the invisible structure of matter.

But I shall not pursue the question as to the proper application of the label ' philosophy ' any further ; for this problem, which is Wittgenstein's problem, clearly turns out to be one of linguistic usage, a pseudo-problem which by now must be rapidly developing into a bore to my audience. But I wish to add a few more words on Plato's theory of Forms or Ideas, or more precisely, on point (6) of the list of historical facts given above.

Plato's theory of the structure of matter can be found in the *Timaeus*. It has at least a superficial similarity with the modern theory of solids which interprets them as crystals. His physical bodies are composed of invisible elementary particles of various shapes, the shapes being responsible for the macroscopic properties of visible matter. The shapes of the elementary particles, in their turn, are determined by the shapes of the plane figures which form their sides. And these plane figures, in their turn, are ultimately all composed of *two* elementary triangles, viz. the half-square (*or isosceles* rectangular) triangle which incorporates the *square root of two*, and the half-equilateral rectangular triangle which incorporates the *square-root of three*, both of them irrationals.

These triangles, in their turn, are described as the copies [1] of unchanging ' Forms ' or ' Ideas ', which means that specifically *geometrical* ' Forms ' are admitted into the company of the Pythagorean *arithmetical* Form-Numbers.

There is little doubt that the motive of this construction is the attempt to solve the crisis of atomism by incorporating the irrationals into the last elements of which the world is built. Once this has been done, the difficulty of the existence of irrational distances is overcome.

But why did Plato choose just these two triangles? I have elsewhere [2] expressed the view, as a conjecture, that Plato believed that all other irrationals might be obtained by adding to the rationals multiples of the square roots of two and three. I now feel quite confident that the crucial passage in the *Timaeus* clearly implies this doctrine (which was mistaken, as Euclid later showed). For in the passage in question, Plato says quite clearly that ' *All* triangles are derived from two, each having a right angle ', going on to specify these two as the half-square and half-equilateral. But this can only mean, in the context, that *all* triangles can be composed by combining these two, a view which is equivalent to the mistaken theory of the relative commensurability of all irrationals with sums of rationals and the square roots of two and three.[3]

But Plato did not pretend that he had a proof of the theory in question. On the contrary, he says that he assumes the two triangles as principles ' in accordance with an account which combines probable conjecture with necessity '. And a little later, after explaining that he takes the half-equilateral triangle as the second of his principles, he says, ' The reason is too long a story ; but if anybody should test this matter, and prove that it has this property ' (I suppose the property that all other triangles can be composed of these two) ' then the prize is his, with all our good will '.[4] The language is somewhat obscure, and no doubt the reason is that Plato lacked a proof of his conjecture

[1] For the process by which the triangles are stamped out of space (the ' mother ') by the ideas (the ' father '), cf. my *Open Society*, note 15 to ch. 3, and the references there given, as well as note 9 to ch. 6.

[2] In the last quoted note

[3] In the note referred to I also conjectured that it was the close approximation of the sum of these two square roots to π which encouraged Plato in his mistaken theory. Although I have no new evidence, I believe that this conjecture is much strengthened by the view that Plato in fact believed in the mistaken theory described here.

[4] The two quotations are from the *Timaeus*, 53c/d and 54a/b

concerning these two triangles, and felt it should be supplied by somebody.

The obscurity of the passage had the strange effect that Plato's quite clearly stated choice of triangles introduce irrationals into his world of Forms seems to have escaped notice, in spite of Plato's emphasis upon the problem in other places. And this fact, in turn, may perhaps explain why Plato's Theory of Forms could appear to Aristotle to be fundamentally the same as the Pythagorean theory of form-numbers,[1] and why Plato's atomism appeared to Aristotle merely

[1] I believe that our consideration may throw some light on the problem of Plato's famous 'two principles'—'The One' and 'The Indeterminate Dyad'. The following interpretation develops a suggestion made by van der Wielen (*De Ideegetallen van Plato*, 1941, p. 132 f.) and brilliantly defended against van der Wielen's own criticism by Ross (Plato's *Theory of Ideas*, p. 201). We assume that the 'Indeterminate Dyad' is a straight line or distance, not to be interpreted as a unit distance, or as having yet been measured at all. We assume that a point (limit, *monas*, 'One') is placed successively in such positions that it divides the Dyad according to the ratio $1 : n$, for any natural number n. Then we can describe the 'generation' of the numbers as follows. For $n = 1$, the Dyad is divided into two parts whose ratio is $1 : 1$. This may be interpreted as the 'generation' of Twoness out of Oneness and the Dyad, since we have divided the Dyad into *two* equal parts. Having thus 'generated' the number 2, we can divide the Dyad according to the ratio $1 : 2$ (and the larger section, as before, according to the ratio $1 : 1$), thus generating *three* equal parts and the number 3; generally, the 'generation' of a number n gives rise to a division of the Dyad in the ratio $1 : n$, and with this, to the 'generation' of the number $n + 1$. (And in each stage intervenes the 'One', the point which introduces a limit or form or measure into the otherwise 'indeterminate' Dyad, afresh, to create the new number; this remark is intended to strengthen Ross' case against van der Wielen's.)

Now it should be noted that this procedure, although it 'generates' (in the first instance, at least) only the series of natural numbers, nevertheless contains a *geometrical* element—the division of a line, first into two equal parts, and then into two parts according to a certain proportion $1 : n$. Both kinds of division are in need of geometrical methods, and the second, more especially, needs a method such as Eudoxus' Theory of Proportions. Now I suggest that Plato began to ask himself why he should not divide the Dyad also in the proportion of $1 : \sqrt{2}$ and of $1 : \sqrt{3}$. This, he must have felt, was a departure from the method by which the natural numbers are generated; it is less 'arithmetical' still, and it needs more specifically 'geometrical' methods. But it would 'generate', in the place of natural numbers, linear elements in the proportion $1 : \sqrt{2}$ and $1 : \sqrt{3}$, which may be identical with the 'atomic lines' (*Metaphysics*, 992a19) from which the atomic triangles are constructed. At the same time, the characterisation of the Dyad as 'indeterminate' would become highly appropriate, in view of the Pythagorean attitude (cf. Philolaos, Diels fragm. 2 and 3) towards the irrational. (Perhaps the name 'The Great and the Small' began to be replaced by 'The Indeterminate Dyad' when irrational proportions were generated in addition to rational ones.)

as a comparatively minor variation of that of Democritus. Aristotle, in spite of his association of arithmetic with the odd and even, and of geometry with the irrational, does not appear to have taken the problem of the irrationals seriously. It appears that he took Plato's reform programme for geometry for granted; it had been partly carried out by Eudoxus before Aristotle entered the Academy, and Aristotle was only superficially interested in mathematics. He never alludes to the inscription on the Academy.

To sum up, it seems probable that Plato's theory of Forms was, like his theory of matter, a re-statement of the theories of his predecessors, the Pythagoreans and Democritus respectively, in the light of his realisation that the existence of irrationals demanded the emancipation of geometry from arithmetic. By encouraging this emancipation, Plato contributed to the development of Euclid's system, the most important and influential deductive theory ever constructed. By his adoption of geometry as the theory of the world, he provided Aristarchus, Newton, and Einstein with their intellectual toolbox. The calamity of Greek atomism was thus transformed into a momentous achievement. But Plato's scientific interests are partly forgotten. The problem-situation in science which gave rise to his philosophical problems is little understood. And his greatest achievement, the geometrical theory of the world, has influenced our world-picture to such an extent that we unconsciously take it for granted.

6

One example never suffices. As my second example, out of a great many interesting possibilities, I choose Kant. His *Critique of Pure Reason* is one of the most difficult books ever written. Kant wrote in undue haste, and about a problem which, I shall try to show, was insoluble. Nevertheless it was not a pseudo-problem, but an in-

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Assuming this view to be correct, we might conjecture that Plato approached slowly (beginning in the *Hippias Major*, and thus long before the *Republic*—as opposed to a remark made by Ross *op. cit.*, top of page 56) to the view that *the irrationals are numbers*, since both the natural numbers and the irrationals are 'generated' by similar and essentially geometric processes. But once this view is reached (and it was first reached, it appears, in the *Epinomis* 990d–e, whether or not this work is Plato's), then even the irrational triangles of the *Timaeus* become 'numbers' (i.e. characterised by numerical, if irrational, propositions). But with this, the peculiar contribution of Plato, and the difference between his and the Pythagorean theory, is liable to become indiscernible; and this may explain why it has been lost sight of, even by Aristotle.

escapable problem which arose out of the contemporary situation of physical theory.

His book was written for people who knew some Newtonian stellar dynamics and who had at least some idea of its history—of Copernicus, Tycho, Brahe, Kepler, and Galileo.

It is perhaps hard for intellectuals of our own day, spoilt and blasé as we are by the spectacle of scientific success, to realise what Newton's theory meant, not just for Kant, but for any eighteenth century thinker. After the unmatched daring with which the Ancients had tackled the riddle of the Universe, there had come a period of long decay, recovery, and then a staggering success. Newton had discovered the long sought secret. His geometrical theory, based on and modelled after Euclid, had been at first received with great misgivings, even by its own originator.[1] The reason was that the gravitational force of attraction was felt to be ' occult ', or at least something which needed an explanation. But although no plausible explanation was found (and Newton scorned recourse to *ad-hoc* hypotheses), all misgivings had disappeared long before Kant made his own important contribution to Newtonian theory, 78 years after the *Principia*.[2] No qualified judge [3] of the situation could doubt any longer that the theory was true. It has been tested by the most precise measurements, and it had always been right. It had led to the prediction of minute deviations from Kepler's laws, and to new discoveries. In a time like ours, when theories come and go like the buses in Piccadilly, and when every schoolboy has heard that Newton has long been superseded by Einstein, it is hard to recapture the sense of conviction which Newton's theory inspired, or the sense of elation, and of liberation. A *unique event* had happened in the history of thought, one which could never be repeated: the first and final discovery of the absolute truth about the universe. An age-old dream had come true. Mankind had obtained *knowledge*, real, certain, indubitable, and demonstrable knowledge—divine *scientia* or *epistēmē*, and not merely *doxa*, human opinion.

Thus for Kant, Newton's theory was simply true, and the belief in its truth remained unshaken for a century after Kant's death. Kant

[1] See Newton's letter to Bentley, 1693.

[2] The so-called Kant-Laplacean Hypothesis published by Kant in 1755.

[3] There had been some very pertinent criticism (especially by Leibniz and Berkeley) but in view of the success of the theory it was—I believe rightly—felt that the critics had somehow missed the point of the theory. We must not forget that even today the theory still stands, with only minor modifications, as an excellent first (or, in view of Kepler, perhaps as a second) approximation.

to the end accepted what he and everybody else took for a fact, the attainment of *scientia* or *epistēmē*. At first he accepted it without question. This state he called his 'dogmatic slumber'. He was roused from it by Hume.

Hume had taught that there could be no such thing as certain knowledge of universal laws, or *epistēmē*; that all we knew was obtained with the help of observation which could be only of particulars, so that our knowledge was uncertain. His arguments were convincing (and he was, of course, right). But here was the fact, or what appeared as a fact—Newton's attainment of *epistēmē*.

Hume roused Kant to the realisation of the near absurdity of what he never doubted to be a fact. Here was a problem which could not be dismissed. How could a man have got hold of such knowledge? Knowledge which was general, precise, mathematical, demonstrable, indubitable, and yet explanatory of observed facts?

Thus arose the central problem of the *Critique*: 'How is pure natural science possible?'. By 'pure natural science'—*scientia, epistēmē*—Kant means, simply, Newton's theory.

Although the *Critique* is badly written, and although it abounds in bad grammar, this problem was not a linguistic puzzle. Here was knowledge. How could we ever attain it? The question was inescapable. But it was also insoluble. For the apparent fact of the attainment of *epistēmē* was no fact. As we now know, or believe, Newton's theory is no more than a marvellous hypothesis, an astonishingly good approximation; unique indeed, but not as divine truth, only as a unique invention of a human genius; not *epistēmē*, but belonging to the realm of *doxa*. With this, Kant's problem, 'How is pure natural science possible', collapses, and the most disturbing of his perplexities disappear.

Kant's proposed solution of his insoluble problem consisted of what he proudly called his 'Copernican Revolution' of the problem of knowledge. Knowledge—*epistēmē*—was possible because we are not passive receptors of sense data, but their active digestors. By digesting and assimilating them, we form and organise them into a Universe. In this process, we impose upon the material presented to our senses the mathematical laws which are part of our digestive and organising mechanism. Thus our intellect does not discover universal laws in nature, but it prescribes its own laws and imposes them upon natures.

This theory is a strange mixture of absurdity and truth. It is as absurd as the mistaken problem it attempts to solve ; for it proves too much, being designed to prove too much. According to Kant's theory, 'pure natural science' is not only *possible* ; although he does not realise this, it becomes the *necessary result* of our mental outfit. For if the fact of our attainment of *epistēmē* can be explained at all by the fact that our intellect legislates for and imposes its own laws upon nature, then the first of these two facts cannot be contingent any more than the second. Thus the problem is no longer how Newton could make his discovery but how everybody else could have failed to make it. How is it that our digestive mechanism did not work much earlier ?

This is a patently absurd consequence of Kant's idea. But to dismiss it offhand, and to dismiss his problem as a pseudo-problem is not good enough. For we can find an element of truth in his idea (and a much needed correction of some Humean views), after reducing his problem to its proper dimensions. His question, we now know, or believe we know, should have been : 'How are successful hypotheses possible ?' And our answer, in the spirit of his Copernican Revolution, might, I suggest, be something like this : Because, as you said, we are not passive receptors of sense data, but active organisms. Because we react to our environment not always merely instinctively, but sometimes consciously and freely. Because we can invent myths, stories, theories ; because we have a thirst for explanation, an insatiable curiosity, a wish to know. Because we not only invent stories and theories, but try them out and see whether they work and how they work. Because by a great effort, by trying hard and erring often, we may sometimes, if we are lucky, succeed in hitting upon a story, an explanation, which 'saves the phenomena' ; perhaps by making up a myth about 'invisibles', such as atoms or gravitational forces, which explain the visible. Because knowledge is an adventure of ideas. These ideas, it is true, are produced by us, and not by the world around us ; they are not merely the traces of repeated sensations or stimuli or what not ; here you were right. But we are more active and free than even you believed ; for similar observations or similar environmental situations do not, as your theory implied, produce similar explanations in different men. Nor is the fact that we originate our theories, and that we attempt to impose them upon the world, an explanation of their success, as you believed. For the overwhelming majority of our theories, of our freely invented ideas, are unsuccessful ; they do not stand up to searching tests, and are discarded as falsified by experience. Only a very few of them succeed, for a time, in the competitive struggle for survival.[1]

7

Few of Kant's successors appear ever to have clearly understood the precise problem-situation which gave rise to his work. There were two such problems for him, Newton's dynamics of the heavens, and the absolute standards of human brotherhood and justice to which the French revolutionaries appealed, or, as Kant puts it, ' the starry heavens above me, and the moral law within me '. But Kant's starry heavens are seldom identified as an allusion to Newton.[2] From Fichte onward,[3] many have copied Kant's ' method ' and the dreadful style of his *Critique*. But most of these imitators have forgotten Kant's original interests and problems, busily trying either to tighten, or else to explain away, the Gordian knot in which Kant, through no fault of his own, had tied himself up.

We must beware of mistaking the well-nigh senseless and pointless subtleties of the imitators for the pressing and genuine problems of the pioneer. We should remember that his problem, although not an empirical one in the ordinary sense, nevertheless turned out, unexpectedly, to be in some sense factual (Kant called such facts ' transcendental '), since it arose from an apparent but non-existent instance of a *scientia* or *epistēmē*. And we should, I submit, seriously consider the suggestion that Kant's answer, in spite of its partial absurdity, contained the nucleus of a philosophy of science.

[1] The ideas of this ' answer ' were elaborated in my *Logik der Forschung* (1935).

[2] That this identification is corrected may be seen from the last ten lines of the penultimate paragraph of the *Critique of Practical Reason*.

[3] Cf. my *Open Society*, note 58 to ch. 12

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